INTERNATIONAL SCIENTIFIC JOURNAL OF BIRUNI
ISSN (E) 2181-2993
Vol. 2, Issue 2. May. (2023)

REKKURENT FORMULALARNING ANIQ INTEGRALLARGA TADBIQI

OLIMBAYEV To'lqin G'ayrat o'g'li
Urganch Davlat Universiteti o'qituvchisi
MADRAXIMOV Temur Rustambekovich
Urganch Davlat Universiteti o'qituvchisi
TO'RAXONOV Islombek Farhodovich
Urganch Davlat Universiteti o'qituvchisi

https://doi.org/10.24412/2181-2993-2023-2-168-172

ANNOTATSIYA

Ushbu maqolada ketma-ketlikning biror hadidan boshlab keyingi barcha hadlari o'zidan oldingi keluvchi hadlari bilan ifodalash mumkin bo'lsa, bunday ketma-ketliklar qaytadigan yoki rekkurent ketma-ketliklar deyiladi. Rekkurent so'zi yunoncha recurrere – qaytmoq so'zidan olingan. Rekkurent formula analizning bir qancha nazariy va amaliy sohalarida ko'p qo'llaniladi. Biz rekkurent formula bilan hisoblanadigan aniq integrallarni ko'rib chiqamiz.

Kalit so'zlar: *Rekkurent, aniq integral, aniqmas integral, Vallis formulasi.*

ABSTRACT

In this article, if all subsequent terms of a sequence starting from one term can be represented by the preceding terms, then such sequences are called recurring or recurrent sequences. The word recurrent is derived from the Greek word recurrere - to return. The recurrent formula is widely used in several theoretical and practical fields of analysis. We will consider definite integrals calculated by the recurrent formula.

***Key words**: Recurrent, definite integral, indefinite integral, Vallis formula.*

Ma'lumki ketma-ketlikning biror hadidan boshlab keyingi barcha hadlari o'zidan oldingi keluvchi hadlari bilan ifodalash mumkin bo'lsa, bunday ketma-ketliklar qaytadigan yoki rekkurent ketma-ketliklar deyiladi. Rekkurent so'zi yunoncha *recurrere* – qaytmoq so'zidan olingan. Rekkurent formula analizning bir qancha nazariy va amaliy sohalarida ko'p qo'llaniladi. Biz rekkurent formula bilan hisoblanadigan aniq integrallarni ko'rib chiqamiz.

1-Misol: Quyidagi aniq integralni hisoblang[1].

$$J_m = \int_0^{\frac{\pi}{2}} \sin^m x dx \quad J_m' = \int_0^{\frac{\pi}{2}} \cos^m x dx$$

Bu integrallarni hisoblash uchun bo'laklab integrallash formulasidan foydalanamiz:

$$J_m=\int_0^{\frac{\pi}{2}}\sin^m xdx=\int_0^{\frac{\pi}{2}}\sin^{m-1}xd(-\cos x)=-\sin^{m-1}x\cos x\Big|_0^{\frac{\pi}{2}}+(m+1)\int_0^{\frac{\pi}{2}}\sin^{m-2}x\cos^2 xdx=$$

$$=-\sin^{m-1}x\cos x\Big|_0^{\frac{\pi}{2}}+(m-1)\int_0^{\frac{\pi}{2}}\sin^{m-2}xdx-(m-1)\int_0^{\frac{\pi}{2}}\sin^m xdx$$

$J_{m-2}=\int_0^{\frac{\pi}{2}}\sin^{m-2}xdx$ ekanligidan $J_m=\dfrac{m-1}{m}J_{m-2}$ bo‘lishi kelib chiqadi.

Agar $m=2n$ bo‘lsa quyidagiga ega bo‘lamiz:

$$J_{2n}=\int_0^{\frac{\pi}{2}}\sin^{2n}xdx=\frac{(2n-1)\cdot(2n-3)\cdot\ldots\cdot3\cdot1}{2n\cdot(2n-2)\cdot\ldots\cdot4\cdot2}\cdot\frac{\pi}{2}$$

$m=2n+1$ bo‘lsa,

$$J_{2n+1}=\int_0^{\frac{\pi}{2}}\sin^{2n+1}xdx=\frac{2n\cdot(2n-2)\cdot\ldots\cdot4\cdot2}{(2n+1)\cdot(2n-1)\cdot\ldots\cdot3\cdot1}$$

Bundan quyidagi rekkurent formulaga ega bo‘lamiz:

$$J_m=\int_0^{\frac{\pi}{2}}\sin^m xdx=\int_0^{\frac{\pi}{2}}\cos^m xdx=\begin{cases}\dfrac{(m-1)!!}{m!!}\cdot\dfrac{\pi}{2} & agar\ \ m-juft\,bo'lsa\\ \dfrac{(m-1)!!}{m!!} & agar\ \ m-toq\,bo'lsa\end{cases}$$

2-misol: Aniq integralni hisoblang: $H_{k,m}=\int_0^1 x^k\ln^m xdx$ $\quad bu\ yerda\ \ k>0\ \ \ m\in N$

Bo‘laklab integraldan quyidagini topamiz:

$$\int_0^1 x^k\ln^m xdx=\frac{1}{k+1}\cdot x^{k+1}\ln^m x\Big|_0^1-\frac{m}{k+1}\int_0^1 x^k\ln^{k-1}xdx=\frac{1}{k+1}\cdot x^{k+1}\ln^m x\Big|_0^1=0$$

ekanligidan $H_{k,m-1}=\int_0^1 x^k\ln^{m-1}xdx$

ekanini e’tiborga olsak, quyidagi rekkurent formulani hosil qilamiz:

$$H_{k,m}=-\frac{m}{k+1}\cdot H_{k,m-1}$$

bundan quyidagi formulani hosil qilamiz:

$$H_{k,l}=(-1)^m\frac{m!}{(k+1)^{m+1}}$$

3-misol : $I=\int_0^1 (1-x)^p x^q dx$ aniqmas integral mavzusida quyidagi fomulani isbotlagan edik[2]:

$$\int (1-x)^p x^q dx = \frac{(1-x)^p x^{q+1}}{p+q+1} + \frac{p}{p+q+1}\int (1-x)^{p-1} x^q dx$$

bo‘laklab integrallashdan va $\frac{(1-x)^p x^{q+1}}{p+q+1}\Big|_0^1 = 0$ ekanligidan

$I_{p,q} = \int_0^1 (1-x)^p x^q dx = \frac{p}{p+q+1}\cdot \int_0^1 (1-x)^{p-1} x^q dx$, $I_{p-1,q} = \int_0^1 (1-x)^{p-1} x^q dx$ bo‘lganidan

$I_{p,q} = \frac{p}{p+q+1}\cdot I_{p-1,q}$ rekkurent formulani hosil qilamiz:

bundan quyidagi formula hosil bo‘ladi:

$$\int_0^1 (1-x)^p x^q dx = \frac{p!q!}{(p+q+1)!}$$

4-misol: Aniq integralni hisoblang: $\int_0^{\frac{\pi}{2}} \frac{\sin(2m-1)x}{\sin x} dx$

Buning uchun $\int_a^b \sin x dx$ ni aniq integral tarifi yordamida isbotlaymiz

$[a;b]$ segmentni teng n ta bo‘lakga bo‘lib,integral yig’indi tuzib olamiz

$\Delta x_k = \frac{b-a}{n}$ va $a>b$ shartdan

$$\delta_m = \Delta x_k \sum_{k=1}^{n} \sin(a + k\Delta x_k)$$

ifodani quyidagicha o‘zgartiramiz:

$$\sum_{k=1}^{m} \sin(a+k\Delta x_k) = \frac{1}{2\sin\frac{\Delta x_k}{2}} \cdot \sum_{k=1}^{n} 2\sin(a+k\Delta x_k)\sin\frac{\Delta x_k}{2} =$$

$$= \frac{1}{2\sin\frac{\Delta x}{2}} \cdot \sum_{k=1}^{n}\left[\cos\left(a+k-\frac{1}{2}\cdot\Delta x_k\right) - \cos\left(a+k+\frac{1}{2}\cdot\Delta x_k\right)\right] = \frac{\cos\left(a+\frac{1}{2}\Delta x_k\right) - \cos\left(a+n+\frac{1}{2}\Delta x_k\right)}{2\sin\left(\frac{\Delta x_k}{2}\right)}$$

Bundan

$$\delta_n = \frac{\frac{h}{2}}{\sin\left(\frac{h}{2}\right)} \cdot \left[\cos\left(a+\frac{1}{2}\Delta x_k\right) - \cos\left(b+\frac{1}{2}\Delta x_k\right)\right]$$

Demak, $\int_a^b \sin x dx = \lim_{\Delta x_k \to 0} \frac{\frac{\Delta x_k}{2}}{\sin \frac{\Delta x_k}{2}} \left[\cos\left(a + \frac{1}{2}\Delta x_k\right) - \cos\left(b + \frac{1}{2}\Delta x_k\right) \right] = \cos a - \cos b$

Shu usul bilan quyidagi formulani hosil qilamiz:

$$\sum_{i=1}^{n} \cos(a + i\Delta x_k) = \frac{\sin\left(a + n + \frac{1}{2}\Delta x_k\right) - \sin\left(a + \frac{1}{2}\Delta x_k\right)}{2\sin\frac{\Delta x_k}{2}}.$$

Agar biz $a = 0$, $\Delta x_k = 2x$ va $n = m - 1$ desak quyidagiga ega bo‘lamiz:

$$\frac{\sin(2m-1)x}{2\sin x} = \frac{1}{2} + \sum_{i=1}^{n} \cos 2ix$$

Aniq integraldan ushbu ajoyib tenglikka ega bo‘lamiz:

$$\int_0^{\frac{\pi}{2}} \frac{\sin(2m-1)x}{\sin x} dx = \frac{\pi}{2}$$

Bu formulalar murakkab integralllarni hisoblashni osonlashtiradi.

Ma’lumki, $0 < x < \frac{\pi}{2}$ bo`lganda $\sin^{2n+1} x < \sin^{2n} x < \sin^{2n-1} x \qquad (n = 1,2,3,...)$

tengsizliklar o`rinli bo`ladi. Bu tengsizliklarni $[0, \frac{\pi}{2}]$ oraliq bo`yicha integrallab,

$\int_0^{\frac{\pi}{2}} \sin^{2n+1} xdx < \int_0^{\frac{\pi}{2}} \sin^{2n} xdx < \int_0^{\frac{\pi}{2}} \sin^{2n-1} xdx,$

so`ngra 1-misolda keltirilgan formulalardan foydalanib topamiz:

$$\frac{(2n)!!}{(2n+1)!!} < \frac{(2n-1)!!}{(2n)!!} \cdot \frac{\pi}{2} < \frac{(2n-2)!!}{(2n-1)!!}.$$

Bu tengsizliklardan

$$\left(\frac{(2n)!!}{(2n-1)!!} \right)^2 \cdot \frac{1}{2n+1} < \frac{\pi}{2} < \left(\frac{(2n)!!}{(2n-1)!!} \right)^2 \cdot \frac{1}{2n}$$

bo`lishi kelib chiqadi.

Keyingi tengsizliklardan topamiz:

$$\frac{\pi}{2} = \lim_{n \to \infty} \frac{1}{2n+1} \left[\frac{(2n)!!}{(2n-1)!!} \right]^2. \qquad (6)$$

(6) formula Vallis formulasi deyiladi.

Vallis formulasidan quyidagicha hulosaga kelish mumkin: (6) tenglikning chap tomonidagi ketma-ketlikning barcha hadlari ratsional bo‘lgani uchun, irratsional songa intiluvchi ratsional ketma-ketlik mavjud.

Mustaqil yechish uchun misollar.

1. $\int_0^{\frac{\pi}{2}} \cos^m x \cdot \cos(m+2)x dx$
2. $\int_0^{\frac{\pi}{2}} \sin^m x \cdot \cos(m+2)x dx$
3. $\int_0^{a} (a^2 - x^2)^n dx$
4. $\int_0^{\pi} \frac{\sin nx}{\sin x} dx$
5. $\int_0^{\pi} \cos^n x \cdot \cos nx dx$
6. $\int_0^{2\pi} e^{-ax} \cos^{2n} x dx$
7. $\int_0^{\frac{\pi}{2}} \ln \cos x \cdot \cos 2nx dx$
8. $\int_0^{1} \frac{x^n}{\sqrt{1-x^2}} dx$

ADABIYOTLAR (REFERENCES)

1. Azlarov T, Mansurov H. «Matematik analiz» 1-qism Toshkent O’qituvchi 2005 y.
2. Г.Полиа, Г.Сеге «Задачи и теоремы из анализа» I-часть Москва «НАУКА» 1978 г.
3. R. Madrahimov, J. Abdullayev, N. Kamalov “Masala qanday yechiladi” Urganch-2013